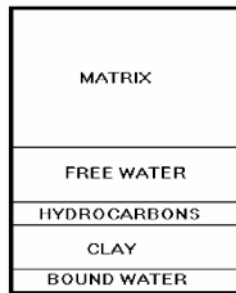


FORMAL DUAL FORMATION MODEL THEORY

The Dual Formation Model provides the ideal logging theoretic setting for the volume fractions of program LVPM as it is operated in “HET” mode. **Constraints** are placed on these fundamental volume fractions and The Good Doctor, unmasked as an inveterate and incurable symbol pusher, provides the rasion d’tre for the term “Dual”, as in Dual Formation Model or in Dual Water Model.

BASIC VOLUMETRICS



In the dual formation model (DFM), the representative unit volume of earth's formations (Figure A1) consists of five volume fractions: (1) hydrocarbons (Φ_h), (2) free water (Φ_f), bound water (Φ_b), (4) clay (V_{cl}), and (5) matrix (V_{ma}). These volume fractions satisfy the fundamental relationship

$$(A1) \quad \Phi_h + \Phi_f + \Phi_b + V_{cl} + V_{ma} = 1$$

All these volume fractions have positive definite values between 0 and 1. By definition, the total porosity (Φ_t), is given by

FIGURE A1: DFM VOLUME FRACTIONS

$$(A2) \quad \Phi_t = \Phi_h + \Phi_f + \Phi_b$$

Notice that Φ_t includes non-fluid material (Φ_b). Neutron based logging tools and chlorine tools tend to respond to Φ_t because they respond to hydrogen and chlorine regardless of where they are chemically or physically located.

Several comments can be made to better understand the scope of these volume fractions. Clean, permeable formations (sand/lime/dolomite) have free water with salinity c_{wf} and shales have bound water with salinity c_{wb} . Bound water content in clays decreases slowly with depth: over several hundred feet this variation is usually negligible. The observed salinity of bound water (c_{wb}) is low and nearly constant. It is doubtful that c_{wb} could ever exceed approximately 0.015 - there is room electrically for Na in the outer Helmholtz plane, but not for Cl. That is, almost all waters associated with clay minerals are chlorine free. Bound water does not include the irreducible water held in place by capillary and surface tension forces. The irreducible water has the same electrical properties as the free water. Bound water is essentially immobile - it cannot be displaced by ordinary chemical or

physical means such as by hydrocarbon or injection fluids. V_{cl} is identified with dry clay, i.e. all HOH groups removed. So-called silt is lumped into V_{ma} . Silt is a very fine-grained material that is predominantly quartz, but may include feldspar, calcite, and other minerals. Silts have neutron and density properties similar to quartz. Shales are clays, water, and silts. By definition, the shale volume (V_{sh}) is given by

$$(A3) \quad V_{sh} = \Phi_b + V_{cl}$$

Notice that when V_{sh} approaches 1, Φ_h , Φ_f , and V_{ma} all approach 0; i.e., 100% shale contains no hydrocarbons, free water, or clean matrix components.

The following saturations are introduced for the free, bound, and hydrocarbon components:

$$(A4) \quad S_{wf} = \Phi_f / \Phi_t \quad S_{wb} = \Phi_b / \Phi_t \quad S_{hy} = \Phi_h / \Phi_t$$

If we define the total water saturation (S_{wt}) by

$$(A5) \quad S_{wt} = S_{wf} + S_{wb}$$

then combination of (2) and (4) shows that

$$(A6) \quad S_{hy} + S_{wt} = 1 \quad S_{hy} = 1 - S_{wt}$$

Note carefully that S_{wt} includes the bound water saturation, S_{wb} - it is not S_w , the classical water saturation. S_{wt} can never be less than S_{wb} . Combining (5) and (6) shows that

$$(A7) \quad S_{wf} + S_{wb} + S_{hy} = 1$$

Equations (1) and (7) are often confused for one another; the former involves volume fractions, while the latter involves saturations which are ratios of volume fractions.

The effective porosity (Φ_e) is defined as the liquid portion of Φ_t :

$$(A8) \quad \Phi_e = \Phi_h + \Phi_f$$

Equations (1), (3) and (8) can be combined into another statement of the fundamental relationship for volume fractions:

$$(A9) \quad \Phi_e + V_{sh} + V_{ma} = 1$$

DUAL FORMATION MODEL ANSATZ

The actual dual formation model attempts to describe real earth formations by placing restrictions on the relative proportions that the volume fractions Φ_h , Φ_f , Φ_b , V_{cl} , and V_{ma} can take in comprising shaly formations. The basic DFM assumption is that two fluid-matrix systems are present and their volume fractions remain proportional to one another. Specifically, Φ_b is to V_{cl} as Φ_e is to V_{ma} :

$$(A10) \quad \Phi_b / V_{cl} = \Phi_e / V_{ma}$$

Since $\Phi_e = \Phi_h + \Phi_f$, this model assumption directly involves all five DFM volume fractions. Using equations (1-4) and (8), this becomes successively:

$$(A11) \quad \begin{aligned} \Phi_b / V_{cl} &= (\Phi_t - \Phi_b) / V_{ma} \\ \Phi_b \cdot V_{ma} &= \Phi_t \cdot V_{cl} - \Phi_b \cdot V_{cl} \\ S_{wb} &= V_{cl} / (V_{cl} + V_{ma}) \\ S_{wb} &= (V_{sh} - \Phi_b) / (1 - \Phi_t) \\ S_{wb} \cdot (1 - \Phi_t) &= V_{sh} - \Phi_b \end{aligned}$$

$$(A12) \quad S_{wb} = V_{sh}$$

The bound water saturation is numerically equal to the shale volume fraction. Mathematically, equations (A10 and A12) represent a constraint that the five fundamental volume fractions must obey: it will allow a reduction in the number of independent primary variables to just three (Φ_t , V_{sh} , S_{wt}) in order to describe shaly sand formations. Generalizations of A10 - A12 are being sought that will permit an even more realistic description of shales and clay minerals.

It is interesting to compute the clay porosity (Φ_c) in the DFM: it is the volume fraction of bound water divided by the total volume fraction of shale:

$$\Phi_{cl} = \Phi_b / V_{sh} = S_{wb} \Phi_t / V_{sh} = \Phi_t !!!$$

THE FIVE DFM STANDARD VOLUME FRACTIONS IN TERMS OF Φ_t , V_{sh} , AND S_{wt}

From equations (4) and (6), one finds

$$(A14) \quad \Phi_{hy} = \Phi_t S_{hy} = \Phi_t (1 - S_{wt}).$$

Observe that this equation does not depend on (10) or (11); also, Φ_h does not directly depend on V_{sh} , as do the other four volume fractions.

From equations (4), (5), and (10), we have:

$$\Phi_f = \Phi_t S_{wf} = \Phi_t (S_{wt} - S_{wb})$$

(A15)

$$\Phi_f = \Phi_t (S_{wt} - V_{sh})$$

Both Φ_h and Φ_f directly depend on S_{wt} , although their sum does not.

From equations (4) and (11),

$$\Phi_b = \Phi_t V_{sh}.$$

(A16)

Combining equations (3) and (15), one obtains

$$V_{cl} = V_{sh} - \Phi_b = V_{sh} - V_{sh} \Phi_t$$

(A17)

$$V_{cl} = V_{sh}(1 - \Phi_t)$$

Formulas (15-16) offer some insight into the description of shale within the context of the dual formation model. Equation (16) predicts that the ratio of V_{cl} to V_{sh} decreases as Φ_t increases. Suppose we take the maximum total porosity as 0.50. Then a 100% dead shale with $V_{sh} = 1$ has $V_{cl} = \Phi_b = 0.50$.

Finally, by combining (1), (3), (13), and (14), we obtain

$$\begin{aligned} V_{ma} &= 1 - V_{cl} - \Phi_b - \Phi_{hy} - \Phi_f \\ &= 1 - V_{sh} - \Phi_{hy} - \Phi_f \\ &= 1 - V_{sh} - \Phi_t (1 - S_{wt}) - \Phi_t (S_{wt} - V_{sh}) \\ &= 1 - V_{sh} - \Phi_t (1 - V_{sh}) \end{aligned}$$

(A18)

$$V_{ma} = (1 - V_{sh})(1 - \Phi_t)$$

(A19)

Equations 13-17 express the five fundamental formation volume fractions in terms of only Φ_t , V_{sh} , and S_{wt} . Observe that Φ_b , V_{cl} , and V_{ma} do not directly depend on S_{wt} . Thus we can obtain Φ_b , V_{cl} , V_{ma} , and Φ_e without S_{wt} .

SECONDARY VOLUMETRIC RELATIONSHIPS

Other formulas are of interest. From equations (8), (13), and (14) an expression for effective porosity can be derived:

$$\Phi_e = \Phi_{hy} + \Phi_f = \Phi_t(1 - S_{wt}) + \Phi_t(S_{wt} - V_{sh})$$

(A20)

$$\Phi_e = \Phi_t(1 - V_{sh}).$$

As remarked earlier, Φ_e does not directly depend on S_{wt} .

Combining equations (5) and (10) shows that

$$(A21) \quad S_{wt} = S_{wf} + V_{sh}$$

As above, S_{wt} cannot be less than V_{sh} ; S_{wt} varies from V_{sh} to 1.

The traditional or classical water saturation S_w , defined by

$$(A22) \quad S_w = \Phi_f / \Phi_e = \Phi_f / (\Phi_f + \Phi_{hy})$$

is also expressible in terms of our primary variables by using equations (13) and (14):

$$S_w = [\Phi_t(S_{wt} - V_{sh})] / [\Phi_t S_{wt} - \Phi_t V_{sh} + \Phi_t - \Phi_t S_{wt}]$$

$$S_w = \frac{\Phi_t(S_{wt} - V_{sh})}{[\Phi_t S_{wt} - \Phi_t V_{sh} + \Phi_t - \Phi_t S_{wt}]}, \text{ or finally,}$$

(A23)

$$S_w = \frac{(S_{wt} - V_{sh})}{(1 - V_{sh})} = \frac{S_{wf}}{(1 - V_{sh})}.$$

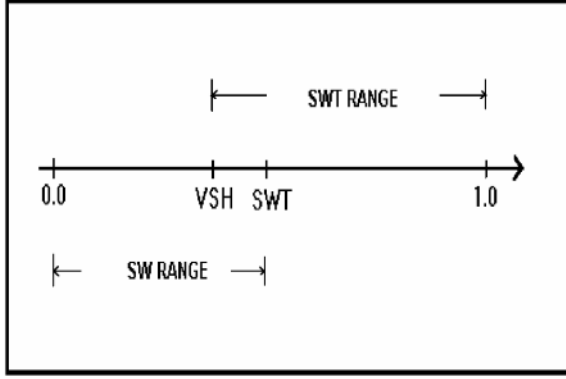


FIGURE A2: RANGES FOR SWT AND SW

Hence, as S_w varies from 0 to 1, S_{wt} varies from V_{sh} to 1. S_w is never larger than S_{wt} . These rules are pictured in the figure at the left. Traditional pathological behavior in S_w , when $V_{sh}=1$ or when $\Phi_f=\Phi_e=0$, is skirted in the dual formation model by using S_{wt} , not S_w , together with Φ_h , Φ_f , Φ_b , V_{cl} , and V_{ma} to describe earth formations.

Comparison of equations (19) and (21) shows that S_w is not equal to S_{wf} , but in fact,

$$S_{wf} = (1 - V_{sh})S_w.$$

When $V_{sh} = 0$, then $S_{wt} = S_{wf} = S_w$.

It is sometimes useful to express Φ_b in terms of Φ_t and V_{cl} ; from equations (3) and (15):

$$\Phi_b = \Phi_t V_{sh} = \Phi_t (V_{cl} + \Phi_b)$$

$$\Phi_b - \Phi_t \Phi_b = \Phi_t V_{cl}$$

$$(A24) \quad \Phi_b = \frac{\Phi_t V_{cl}}{1 - \Phi_t}$$

Equation (16) is sometimes written as

$$(A25) \quad V_{sh} = V_{cl} / (1 - \Phi_t)$$

which allows calculation of V_{sh} from V_{cl} and Φ_t . As V_{sh} varies from 0 to 1, V_{cl} varies from 0 to $1 - \Phi_t$.

A standard **producibility condition** for shaly reservoirs is

$$(A26) \quad V_{sh} < 2\Phi_t / (1 + 2\Phi_t)$$

i.e., if this condition is satisfied, the reservoir is said to be producible. Use of equation (18) can elucidate this condition:

$$\begin{aligned} V_{sh} &< 2\Phi_t / (2\Phi_t + 1) \\ 2\Phi_t V_{sh} + V_{sh} &< 2\Phi_t \\ V_{sh} &< 2\Phi_t (1 - V_{sh}) \end{aligned}$$

$$(A27) \quad V_{sh} < 2\Phi_e$$

Thus the break even point of producibility occurs when V_{sh} is twice the effective porosity.

A formula similar to the basic DFM assumption (10) can be derived. From (23) and (1):

$$\begin{aligned} V_{cl} / V_{sh} &= 1 - \Phi_t = V_{cl} + V_{ma} \\ V_{cl} &= V_{cl} V_{sh} + V_{ma} V_{sh} \\ V_{cl}(1 - V_{sh}) &= V_{ma} V_{sh} \\ V_{cl} / V_{sh} &= V_{ma} / (1 - V_{sh}) \end{aligned}$$

$$(A28) \quad V_{cl} / (\Phi_b + V_{cl}) = V_{ma} / (\Phi_e + V_{ma})$$

Finally, an important practical relationship between S_{wt} and S_w can be established. From equation (8),

$$\begin{aligned} \Phi_{hy} &= \Phi_e - \Phi_f = \Phi_e [(\Phi_e - \Phi_f) / \Phi_e] \\ \Phi_{hy} &= \Phi_e (1 - \Phi_f / \Phi_e) \\ \Phi_{hy} &= \Phi_e (1 - S_w) \end{aligned}$$

$$(A29) \quad \Phi_t (1 - S_{wt}) = \Phi_e (1 - S_w).$$